King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering

Information and Computer Science Department

ICS 253: Discrete Structures I

Summer Semester 2012-2013

Major Exam #2, Saturday July 13, 2013.

Name:

ID#:

**Instructions**:

1. This exam consists of **seven** pages, including this page, containing **three** questions. An additional helping sheet is also attached.
2. You have to answer all **three** questions.
3. The exam is closed book and closed notes. No calculators or any helping aides are allowed. Make sure you turn off your mobile phone and keep it in your pocket if you have one.
4. The questions are **NOT equally weighed**. Some questions count for more points than others.
5. The maximum number of points for this exam is **100**.
6. You have exactly **90** minutes to finish the exam.
7. Make sure your answers are **readable**.
8. If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order for me not to miss grading it.

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| Question Number | Maximum # of Points | Earned Points |
| 1 | **35** |  |
| 2 | **45** |  |
| 3 | **20** |  |
| **Total** | **100** |  |

1. **(35 points) Basic Structures: Sets, Functions, Sequences and Sums.**
	1. (10 points) Draw the graph of the function

$f\left(x\right)=\left⌈\frac{x}{2}\right⌉+\left⌊x\right⌋$ where $-4\leq x\leq 4$



* 1. (7 points) Find $\bigcup\_{i=1}^{\infty }A\_{i}$ and $\bigcap\_{i=1}^{\infty }A\_{i}$ if for every positive integer *i*,

*Ai* = (-*i*, *i*), that is, the set of real numbers *x* with -*i* < *x* < *i*.

1. (8 points) Give an example of two uncountable sets A and B such that $A∩B$ is
	* 1. empty.
		2. finite but not empty.
		3. countably infinite.
		4. uncountable.
2. (10 points) Compute the following summation:

$$\sum\_{i=7}^{10}\sum\_{j=1}^{8}\left(2^{j}-j\left(3^{i}\right)\right)$$

1. **(45 points) Induction and Recursion**
	1. (5 points) Give a recursive definition of $a\_{n}=2n+1 n=1,2,3,…$
	2. (10 points) Prove that 12 − 22 + 32 −· · ·+ (−1)*n*−1*n*2 = $ \frac{\left(-1\right)^{n-1}n\left(n+1\right)}{2}$ whenever *n* is a positive integer.
	3. (20 points) Answer the following questions related to forming different values of stamps:
		1. (5 points) Which amounts of postage can be formed using only 5-cent and 7-cent stamps? Make sure your statement is of the form: All values greater than or equal to *n* can be formed using only 5-cent and 7-cent stamps.
		2. (15 points) Prove the conjecture you made using strong induction.
	4. (10 points) The set of leaves and the set of internal vertices of a full binary tree can be defined recursively.

*Basis step*: The root *r* is a leaf of the full binary tree with exactly one vertex *r*. This tree has no internal vertices.

*Recursive step:* The set of leaves of the tree *T* = *T*1 · *T*2 is the union of the sets of leaves of *T*1 and of *T*2. The internal vertices of *T* are the root *r* of *T* and the union of the set of internal vertices of *T*1 and the set of internal vertices of *T*2.

Use structural induction to show that *l(T )*, the number of leaves of a full binary tree *T* , is 1 more than *i(T )*, the number of internal vertices of *T* .

1. **(20 points) Counting**
	1. (5 points) Derive the number of one-to-one functions that are from a set with 5 elements to a set with 10 elements.
	2. (5 points) Derive the number of strings of eight English letters that contain at least one vowel, if letters can be repeated. Note: There are 5 vowels in English.
2. (10 points) An arm wrestler is the champion for a period of 75 hours. (Here, by an hour, we mean a period starting from an exact hour, such as 1 p.m., until the next hour.) The arm wrestler had at least one match an hour, but no more than 125 total matches. Show that there is a period of consecutive hours during which the arm wrestler had exactly 24 matches.

**Some Useful Formulas**

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|  | Addition |  | Modus Tollens |
|  | Simplification |  | Hypothetical syllogism |
|  | Conjunction |  | Disjunctive syllogism |
|  | Modus Ponens |  | Resolution |

